A CLASS OF CONTENT-PROBLEMS FOR HIGH-SCHOOL ALGEBRA

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Some fifteen years ago the doctrine of correlation was just beginning in the elementary school. Practical teachers in the grades then looked upon the advocates of the doctrine as faddists, or *visionaires*. First attempts to practice the doctrine were half-hearted, sporadic, and poorly planned. But the great influx of new material was threatening to submerge and to disrupt elementary-school curricula. School administrators felt that something must be done to avert disaster, and the most advanced of them began urging upon teachers the imperative need of studying into the larger unities of the rich supply of materials that had accumulated. Slowly the teachers took up the work. The first efforts to correlate the raw materials of education around a central idea were feeble, and but little better than travesties on the idea. School work became spotted. One idea was well—even brilliantly—worked out with the class; then there was rambling, time-filling, and wool-gathering. The new attempts became the target of sharp criticism, then of keen ridicule. But, by those who tested the doctrine, the brilliant spots were felt to be better than a dead level of stale insipidity. The patient maturing of the practical phases of the doctrine, and a fuller trying-out under the light of past failures, have at length secured the general, though sometimes tacit, adoption of the doctrine of correlation in all the leading elementary schools of our country. Nowadays every new subject that knocks for admission into the elementary school, no less than every one of the old pre-emptors of the ground, must justify its claims on grounds of its bearings on the all-round work of the school. All of these matters may serve well for enlightenment and encouragement in the struggle with intrenched tradition and buttressed conservatism in the secondary school.

As the field of battle for better teaching shifts from the elementary

to the secondary domain, it is only natural that the leaders of the reform should resort to the same line of tactics that has proved successful in the field already taken. Accordingly, the doctrines of correlation are recommended to secondary teachers who are seeking improvement. The danger which is always present at this point is that the recommendation will be construed to mean the same phase of correlation as has wrought the improvement in elementary teaching. But it will hardly be expected that the same breadth of correlation where *extensive* study is the characteristic feature of the pupil's effort can be successfully attempted where *intensive* work is wanted from the student, as is the case in the secondary school.

The law that what is best for the adult is not necessarily best for the child, which has protected elementary programs from the *ex cathedra* utterances of college teachers, may well be stated now and then the other way around. This shifts the emphasis and materially alters the meaning. What is best for the child is not necessarily best for adults. As an illustraton, it may be cited that, while an adult can very profitably work under a *remote motive*, a child needs an immediate motive to draw forth his best effort. But the change from the mental traits of childhood to those of adulthood is a growth. No one can put his finger on any stage of the school career and say *here* is an abrupt break in continuity of growth. By the beginning of the high-school period, however, there has been a very considerable accumulation of modifications, and the significant question for the practical high-school man is: What do these changes mean as to the necessary modifications of, or deviations from, elementary-school procedure? Programs of study, textbooks, and discussions all seem to pre-suppose only *difference* of procedure. However unphilosophical this consensus may be, it can nevertheless be explained, if not justified, historically. The vital question is: How much of likeness and how much of difference should characterize elementary and secondary work? It is this unanswered question which makes the chief trouble for mathematics.

It has been customary of late to argue and to work for an all-embracing correlation of early high-school mathematics with science and with the quantitative phase of the pupil's life. This sort of correlation, though ideal, is very seriously hampered both by the

present order of science subjects in the curriculum and by the extreme difficulty of so organizing the pupil's out-of-school interests as to accomplish the ends of mathematical study. The difficulty just mentioned is, nevertheless, being attacked, and progress is being made in overcoming it; but progress in this direction is necessarily slow.

Some have narrowed the problem of correlation down to a unification of *mathematics* and *physics* in teaching. But even this phase of the problem of securing a continuity and homogeneity to the pupil's school life, commendable as it is, is hampered both by the present arrangement of subjects in the program and by the difficulty of procuring teachers that are able to handle well both physics and mathematics. In the May number of this journal Mr. Moore narrows the problem of correlation still more closely. As a first stage in progress toward a full-orbed correlation of all cognate quantitive concerns of the pupil, his suggestion is to correlate, or to unify, the *mathematical subjects* into a homogeneous body of truth through the extensive and continuous use of the cross-ruled paper. This plan is eminently practicable. Programs need not be disturbed outside the mathematical subjects, and these disturbances may readily be taken care of by mathematical teachers themselves. There can be no objection to the proposal on grounds of expense, for cross-lined paper can be had at a cost so nominal that pupils themselves can bear the burden. From the numerous and varied uses of the graph that Mr. Moore suggests, it would seem that everyone who desires to assist in improving teaching—and who does not?—could readily find a place to lay hold at once. The paper points out in detail how any analytical operation a high-school pupil is ordinarily called upon to make, as well as many other operations that are too complicated for him analytically, may be simply and vividly put before him graphically by the aid of the cross-ruled paper. If this graphical work be now put into some sort of organic relationship with real problem-work—work that seems real to the pupil—that best of all results of teaching will in good degree be secured, viz., an interest in and love for mathematical thinking. As teachers of mathematics we need to keep continually before our minds the fact that the highest of all educational aims

is neither *learning* nor the mere *power to acquire learning*, but a *real love lo learn.* Interest and purpose are necessary to the attainment of this aim in mathematical study.

It is the demand of the hour for high-school mathematics, and is the spirit of Mr. Moore's paper, that some means must be found of enabling the pupil, as early and as continuously as possible, to realize the worth to him of his mathematical tasks. The problem of arousing and sustaining interest will then be solved. The work must have such content and treatment as to place and keep the pupil where, by his own standards of worth, he may both gauge and approve the merit of what he is required to do. It is not the intention even remotely to imply by the foregoing statement that the teacher is to ask the pupil whether the latter thinks a proposed task is worth while. What is intended is that the nature of the tasks put before the learner shall be such that, no matter how great their difficulty, there will be a steady undertone of assent on the part of the learner that the tasks are worth the effort. This is recognized as essential in other subjects. It must be so recognized in mathematics before its full educational value can be realized.

Youth of the early high-school period are profoundly impressed with "the *go*" and "the *do*" of things. They feel that they have added a cubit to their mental stature when they have learned a new truth about the way man and nature accomplish results. The number of young persons on the bleachers at a high-school game, who are enthusiasts in convincing listeners that they know how to do it is out of all proportion to the number who can actually do it. Some have criticized our school system as a producer of "lip-doers" rather than actual doers. Be this as it may, the normal youth of fourteen to sixteen, or seventeen, loves to learn how the forces of nature are made to do the work of man and of nature. A machine is of interest to him, not as a machine, but because it does something. A large number of our simple machines, utensils, and our commonest apparatus are based upon the laws of parallel forces, and, with very little time and almost no technical introduction, numerous problems, covering a wide range of algebraic theory and technique, might be used to enrich the content of early algebra with conceptional material that makes a general appeal to youth. It is the main purpose of

this paper to show how easy and practicable it is for mathematical teachers to levy tribute from this field.

Two simple principles, or laws, will be needed, and it will be shown here how, in three forty-five minute recitation periods first-year pupils were given a working hold on these laws.

A light, wooden bar, as *ss* in the accompanying figure, was so arranged that it might move easily upward or downward, but could not turn around, when upward and downward forces were applied to it at the pegs, along the bar.[1] Two pegs, one at the middle of the bar and the other a little below it at I, work easily within a slot and prevent the bar from turning. The balls at *s* and *s* could be screwed inward or outward to balance the *unloaded* bar horizontally. Weight enough was then placed at *D* to hold the empty bar suspended. After the balances were adjusted, the class was told to pay no further attention to the weights at *D* and at *ss*. These were merely a part of a device for getting a *weightless bar* (or it may be called a *beam*) that may move easily upward, or downward, without turning.[2]

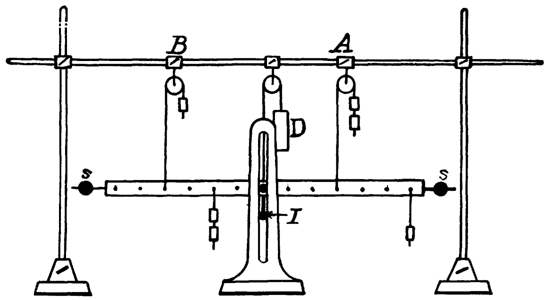

FIG. 1

Light weights could then be hooked on any of the pegs, and, by pulleys, such as those at *A* and *B* in the figure, forces pulling

[1] In this connection compare an article entitled "Algebra Evolved from the Learner's Experience," by Arthur C. Lunn, in *Mathematical Supplement of School Science*, April, 1903; also, "An Algebraic Balance," by E. C. Donecker, in *School Science and Mathematics*, June, 1905; also, "Another Algebraic Balance," by N. J. Lennes, *ibid.*, November, 1905.

[2] The bar and weights used in the apparatus of this paper are parts of an apparatus for experimenting with forces, designated "Mechanical Powers, No. 382," p. 37, of the *Catalogue* of C. H. Stoelting & Co., 31–45 West Randolph street, Chicago, for 1901. With these pieces the work of fitting up the apparatus is almost nothing at all. With only a little more trouble one may provide himself with a home-made device that is good enough. Spools will answer for pulleys.

upward could also be applied at any of the pegs. Any number of pulleys could be quickly hooked over the rod AB. The pegs were equally spaced, and the individual weights were all equal. (Small cans, loaded equally with sand, will answer.) The weight of each was called w or x. Weights pulling upward were written and called positive; those pulling downward, negative.

The class was asked, for each loading, to begin on the left, and to record the circumstances of each loading in the form of an algebraic expression, and to show the value of each expression by an equation. In this expression each force entered as a monomial term with its appropriate sign. For example, with the loading shown in the foregoing cut, the class wrote in their notebooks the following record:

Equation	Result
$+x-2x+2x-x=0$	No movement

For other loadings, the records were as follows:

Equation	Result
$+3x-x-x-x=0,$	No movement
$+2x-3x+2x-2x=-x$	Movement downward
$-4x+2x-2x+5x=+x$	Movement upward
etc.	etc.

A half-dozen to a dozen such records were made, and the "Result" was obtained by noticing the behavior of the bar.

Pupils were next asked to examine their records to see whether, for imaginary loadings, they could tell from the equation what the bar would do. The class was then given such expressions as $+5x-3x+2x-x$; $+4x-x+3x-4x-2x$; etc. Many made correct predictions and records, and the rest soon "caught the scheme." When there was trouble, the apparatus was resorted to.

Pupils were then asked to state what the value of the algebraic sums of all the forces acting on the bar must be *for balance*. With little difficulty, the substance of the following statement was brought out and accepted as the first law for parallel forces:

LAW I: *For balance, the algebraic sum of all the forces acting on a bar, or beam, is equal to zero.*

Beginning at the first peg to the right of the center, the pegs on the right were designated r_1, r_2, r_3, r_4, r_5, and r_6; and those on the

left, l_1, l_2, l_3, l_4, l_5, l_6. Problems were now given of the following types:

With 2 weights, x, at l_3, 1 weight, x, at l_2, 1 weight, x, at r_2, 2 weights, x, at r_3, and an upward force $6x$, at r, write the "record" and the "result." Also give the record and result, if the upward force of $6x$ is replaced by an upward force of $5x$, other weights remaining as above; also, if the $5x$ is replaced by an upward force of $7x$.

Write the appropriate equations and the results, that would be shown by the beam, for the loadings represented by the numbers in each horizontal line of the following table:

Peg........	l_6	l_5	l_4	l_3	l_2	l_1	r_1	r_2	r_3	r_4	r_5	r_6
No.												
1........	$+x$	$+x$	$+x$	$-3x$	$+x$	$-x$	$-x$	$-3x$	$+x$	$+x$	$+x$	$+x$
2........	$-x$	$-x$	$-x$	0	0	$-4x$	$+4x$	$-2x$	$+x$	$-2x$	$-x$	$-x$
3........	$-w$	$+3w$	$-w$	0	0	0	0	0	0	$-2w$	$+w$	$+w$
		etc.							etc.			

Enough of this problem-work was done to make pupils able to use the law easily, to give some ease in adding such monomials as occur in the problems, and to impress pupils with the idea that an equation means the expression of a condition; i. e., it is a record of *what the bar shows by balancing.*

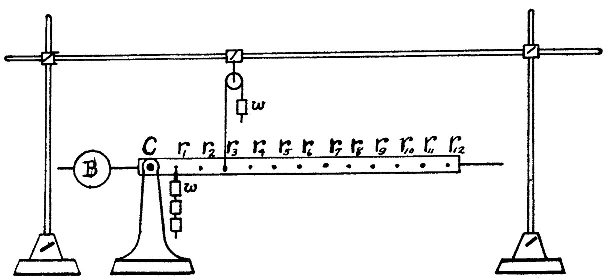

FIG. 2

To obtain a working notion of the second law, the first necessity was to give pupils a mathematical notion of *leverage*, or *turning tendency* (these terms were preferred to the word *moment*). An apparatus like that shown in the adjoining cut was used to give mathematical precision to the notion, after citing the force and leverage required to turn a grindstone, to turn belted pulleys, etc. Here the bar is so arranged that it can turn, but cannot move upward

or downward as a whole, i. e., *in translation*. The bar was balanced empty by the aid of the ball *B*. A weight, x, being at r_1 was seen to turn the right end of the bar downward; while a weight x, pulling upward at r_1 by aid of the pulley, turned the right end upward. Here were two kinds of turning motion, which needed to be distinguished. The kind of turning which the positive force produced, i. e. *anti-clockwise*, was called *positive*, and the opposite kind, i. e., the *clockwise*, was called *negative*. The problem was now to measure and express these *turning tendencies*, or *leverages*, as numbers. The pulley was placed above the peg r_1, and loaded with one weight, w, and weight enough was hung to the peg r_1 to prevent turning. Then the pulley was slipped to the right so that an upward force of w acted on the peg r_2, and enough weights ($2w$) were hung to peg r_1 to prevent turning. The pulley was then slipped along so that an upward force of w could be applied successively to r_3, to r_4, to r_5, etc., and in each case weights enough were hung to r_1 to prevent turning. Calling the distances between the pegs 1 (unity), it was soon found that the number of weights on r_1 was always equal to the product of the number of weights on the pulley-cord by the number of units in the distance from *C* to the peg where the upward force was applied to the bar. Since the bar balanced only when the *positive turning tendency* of the upward force was just equal to the *negative turning tendency* of the downward force, it was clear that the number of weights on peg r_1 *measured the turning tendency* of the upward (positive)force. The distance from *C* to the peg, where any one of the forces, upward or downward, was attached to the bar, was now named the *arm of the force*, and the *turning tendency, or leverage*, of any force was defined *to be the product of number* of units in the force *by the length of its arm*. The class was now told to call the distance between any two adjacent pegs x, and to express all arms in terms of x. Records were then made as was done in deriving Law I. For the loading shown in the cut the record was:

Equation	Result
$(-3w)(x)+(+w)(3x)^*=0$	No turning

* The +'s between the several products were read "and." Thus "minus $3w$ times x and plus w times $3x=-3wx+3wx=0$."

Looking at the bar, the class saw that the first force had a *negative*, and the second a *positive*, turning tendency. Hence, $(-3w)(x)+(+w)(3x)=-3wx+3wx=0$.

Other records with two, or more, forces were as follows:

Equation	Result
(1) $(+3w)(2x)+(-2w)(3x)=$	
$+6wx-6wx=0$	No turning
(2) $(-4w)(2x)+(+w)(2x)+(+2w)(3x)=$	
$-8wx+2wx+6wx=0$	No turning
(3) $(-3w)(2x)+(+w)(2x)+(+2w)(3x)=$	
$-6wx+2wx+6wx=+2wx$ . .	Positive turning
(4) $(-5w)(2x)+(+w)(2x)+(2w)(3x)=$	
$-10wx+2wx+6wx=-2wx$ . .	Negative turning
etc.	etc.

Enough loadings were given to make the pupils sure of their interpretations of signs and turning tendencies. By examining a dozen such records as the above, the fact was soon brought out that for balance of this bar, the algebraic sum of the turning tendencies of all the forces must be equal to zero. This form of the apparatus was then changed for that shown in the following cut.

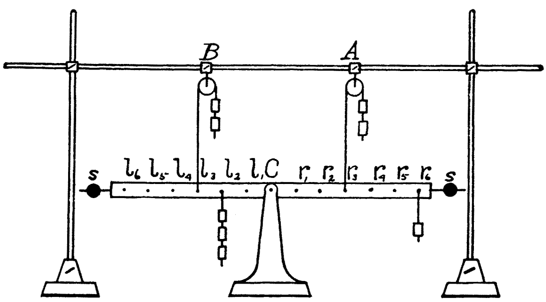

FIG. 3

For this new form of apparatus the following agreements were made and recorded on the blackboard and in the notebooks:

1. *Forces pulling upward on the bar shall be called positive, and written* +; *those pulling downward, negative, and written* −.

2. *Arms measured from C toward the right shall be called positive; toward the left, negative.*

3 *Turning tendencies anti-clockwise shall be called positive; clockwise, negative.*

It is well worth while to give more extended use of the laws of signs in sums and products of signed numbers, than is customary.

A number of loadings were then given and the circumstances recorded thus:

	Equation	Result
(1)	$(+2w)(-3x)+(-3w)(-2x)+(+2w)(+3x)+(-w)(+6x)=-6wx+6wx+6wx-6wx=0$	No turning
(2)	$(+2w)(-3x)+(-3w)(-2x)+(+w)(+3x)+(-w)(+6x)=-6wx+6wx+3wx-6wx=-3wx$	Negative turning
(3)	$(+w)(-5x)+(-2w)(-3x)+(+3w)(+2x)+(-w)(+6x)=-5wx+6wx+6wx-6wx=+wx$	Positive turning
(4)	$(+2w)(-6x)+(-w)(-2x)+(-w)(-x)+(+4w)(+3x)=-12wx+2wx+wx+12wx=+3wx$	Positive turning
(5)	$(+2w)(-5x)+(+w)(-2x)+(+4w)(+x)+(+4w)(+2x)=-10wx-2wx+4wx+8wx=0$	No turning
	etc.	etc.

A number of imaginary loadings were now given, and the class was asked to make records from the equations without reference to the bar. It was soon apparent that everything could be obtained directly from the equation and that the apparatus was no longer necessary. The *second law* of parallel forces was easily derived from an examination of the records, and was stated and written in the notebooks and on the board:

Law II. For balance, the algebraic sum of all turning tendencies must equal zero.

From here on the class was required to take on faith the generalization that, if any sort of bar, beam, or body whatsoever is in equilibrium—i. e., is lying still—under the action of parallel forces, the two laws just derived always hold good.

It was thought well to convince the class that the arms need not be measured always from the middle of the bar, or body; but that they may be measured from *any convenient point along the bar*, and, in particular, from either end. The only requirement was shown to be that, once a point (called the *turning-point*) has been selected to measure arms from, all arms throughout any problem must be measured from this same point. Some of the loadings and exer-

cises given above were solved by taking the turning-point, real or assumed, at some side peg, or at an end peg. The results always agreed with the previous solutions, though the forms of the equations differed with the choice of the turning-point. This is an advantage for teaching, as by a suitable choice of the turning-point the teacher may obtain the form of equation he desires.

By the aid of the two laws of force and of the four axioms of addition, of subtraction, of multiplication, and of division, a variety of practical problems, such as the following, were solved. It seemed well to enhance the pupil's confidence in the utility of the laws by giving first some problems so simple that the pupil could know the result from the outset, and to rise gradually to more difficult problems, whose results could not be foreseen.

SAMPLES OF PROBLEMS AND TYPES OF EQUATIONS THEY CALL FOR

1. A basket, weighing 84 lb., hangs on a stick 6 ft. long at a point $1\frac{1}{2}$ ft. from the end, while it is being carried by two boys, one at each end of the stick. How much does each boy lift?

Taking the turning-point at the middle of the stick, and denoting the weight borne by the boy carrying the short end by x, and that borne by the other boy by y, we write from Law I, $x+y-84=0$, giving $x+y=84$; and from Law II, $(+x)(-3)+(-84)(-1\frac{1}{2})+(+y)(+3)=0$, giving $-3x+3y=-126$.

The solution of the reduced forms calls for standard algebraic methods.

2. Solve a problem like 1, supposing the basket to weigh 60 lb., and to hang at a point 1 ft. from the end of a 4 ft. stick; 2 ft. from the end; 3 ft. from the end.

Taking the turning-point at the middle and denoting the forces as before, the reduced forms of the required equations are:

$$(1)\ \begin{cases} x+y=60 \\ 2x-2y=60 \end{cases} \qquad (2)\ \begin{cases} x+y=60 \\ 2x-2y=0 \end{cases} \qquad (3)\ \begin{cases} x+y=60 \\ 2x-2y=-60 \end{cases}$$

All these are familiar enough as types of linear, simultaneous equations.

3. Two men lifting, one at each end of a stick 8 ft. long, raise a certain weight. How heavy is the weight, and at what point does it hang, if one man lifts 25 lb. and the other 75 lb.?

Solution: From Law I, $+25-w+75=0$, whence $w=100$ lb. Taking the turning-point at the middle of the stick: From Law II $(+25)(-4)+(-w)(-x)+(+75)(+4)=0$ ($-x$ being the arm of the weight); whence $wx=-300+100=-200$. Since $w=100$, $x=-2$. Interpret the result by a sketch.

4. Suppose a bar 10 ft. long, weighing 30 lb., to be used by two men, one grasping it at each end, to carry a load of 170 lb. How many pounds must each man carry, if the load hangs from a point 2 ft. from the front end?

NOTE—The weight, 30 lb., of the bar itself may be treated as a load of 30 lb. hanging to the bar at its middle point.

Solution: Taking the turning-point at the middle of the bar, we have from Law I: $+x-170-30+y=0$, whence $x+y=200$; Law II: $-5x+510-0+5y=0$, whence $-5x+5y=-510$, or $-x+y=-102$.

REMARK.—The zero-term in the second equation arises from the leverage of the weight of the bar, which is 30×0. Here is a chance for a concrete interpretation and an algebraic treatment of the product of *any number multiplied by zero.*

5. A stone slab, weighing 2400 lb., rests with its edge on a crow-bar 6 ft. long, at a point 6 in. from the end which is used as the fulcrum. How many pounds of force must a man lifting at the other end of the bar exert just to raise the stone, (1) omitting the weight of the bar itself? (2) if the bar itself weighs 40 lb.?

Solution of (1): From Law I: $+F-2400+x=0$; or, $F+x=2400$. From Law II: $-3F+6000+3x=0$; or, $-3F+3x=-6000$ (turning-point at middle of bar).

Since the pressure of the support at the fulcrum is usually not wanted, we might take the turning-point *at the fulcrum-end*, whereupon Law II would give: $(+F)(0)+(-2400)(+\frac{1}{2})+(+x)(+6)=0$; or, $6x-1200=0$, whence $x=200$ lb.

REMARK.—This gives another actual practical use of the product of a finite number multiplied by zero. It is also clear that a teacher may adapt the problem to exemplify either the one or the two-unknown mode of solution.

6. Three boys desire to carry a 12-ft. log, weighing 240 lb. Two of the boys lift at the ends of a hand-spike placed cross-wise underneath the log, and the third boy carries the rear end of the log. Where must the hand-spike be placed that all may lift equally?

Solution: Taking the front end of the log as a turning-point, and calling x the distance from the turning-point to the spike, we have from Law I: $2f-240+f=0$; and from Law II: $(+2f)(+x)+(-240)(+6)+(+f)(+12)=0$. From these equations we have: $3f=240$, or $f=80$, and $2fx+12f=1440$. Substituting f, we find $x=3$ ft. Interpret the result.

NOTE.—This is an easy case of algebraic problem in three unknowns. The form of the problem may be modified almost at the pleasure of the teacher, by taking the turning-point at other places along the log.

It is interesting gradually to generalize the foregoing problem by supposing, first, that the length of the log is 6 ft., and the weight 240 lb.; then, that the length is 6 ft, and the weight x lb., the numbeı of boys still being three; and, finally, that the length is 6 ft., the weight is w lb., and the number of boys lifting at the spike is $2n$ (n at either end of the spike, making in all, $2n+1$ boys).

We shall omit the solutions of the following, but they will be found to involve interesting and varied algebraic forms.

7. A steel beam 24 ft. long, and weighing 120 lb. per yard, is being moved by an axle, borne by a pair of wheels placed under it at a point x ft. back of the front end and y ft. in front of the middle point. The rear end is being carried. If the weight carried at the rear end is 200 lb., what must be the values of x and y and of the weight, w, on the axle just to move the beam?

8. How may a railroad rail weighing more than a ton be weighed with a spring balance running only up to 50 pounds?

9. A wheelbarrow is loaded with 45 bricks weighing 6 lb. each. What lifting-force will be needed at the handles just to raise the load, if the hand is $4\frac{1}{2}$ ft. and the center line of the load is 2 ft. back of the center of the wheel?

10. The box of a push-cart is 5 ft. long, and, when full, it holds 660 lb. of earth. The axle is $1\frac{1}{2}$ ft., and the handle-bar is 7 ft., back of the front end of the box. When loaded with earıh, what lifting-force at the handle-bar will just raise the legs from the ground, and what will then be the weight on the axle?

11. If the legs of the push-cart of problem 10 are just under the rear end of the box, what will be the weight on them, and what the weight on the axle, if the cart is standing still on level ground, and loaded with earth?

12. The handle of a suction-pump works against a pin 2 in. from the point where the plunger is attached to it, and the hand seizes the handle

3 ft. from the same point. What lifting-force will be exerted on the plunger, and what will be the pressure on the pin by a downward thrust of 20 lb. at the hand; this being just force enough to work the pump?

13. The arms of a balance are x and y. When a mass w is placed in one scale-pan, 16 lb. placed in the other pan will just balance it. When the mass, w, is placed in the other pan, only 9 lb. are needed to balance it. What is the correct weight of w? What are the lengths of the arms?

Solution: Law II gives $(-w)(-x)+(+F)(0)+(-16)(+6)=0$, or $wx=16y$, for the first weighing; Law II gives $(-9)(-x)+(+F)(0)+(+w)(+y)=0$, or $9x=wy$, for the second weighing.

Dividing, we readily find $w=\pm\sqrt{144}=\pm 12$. Interpret the double sign. Here algebra shows itself liberal and provides for the possibility that the weights may pull either *upward or downward.*

Taking other values instead of the 9 and 16, as 10 and 12, we have a practical problem for the introduction to the study of radicals, for we should then have $w=\pm\sqrt{120}$.

Multiplying the equations, we readily find $\frac{x^2}{y^2}=\frac{16}{9}$, whence $\frac{x}{y}=\pm\frac{3}{4}$. Interpret. Can the plus sign have meaning here? If 10 lb. and 12 lb. were used, we should have $\frac{x}{y}=\sqrt{\frac{10}{12}}$, and again there is a call for radicals.

Common road wagons, carriages, automobiles, floor-joists, bridge-girders, roof-trusses, etc., furnish an abundance of conditions calling for simultaneous equations. The reply that such problems call for much technical knowledge is untenable. Only the two laws already given are needed. Space permits only a few more examples to illustrate how this work may be given a formal turn.

14. A floor-joist lies at rest under the three parallel forces $+x$, $+y$, and -5, whose arms are, $+3$, -2, and $+1$, respectively. Find the strength of the unknown forces x and y.

15. A bridge-girder is at rest under the action of the forces $+x$, -10, $+y$, and -2, with arms, respectively, -4, $+2$, $+5$, and $+11$. Find x and y.

When quadratics are wanted, such problems as this will answer:

16. A roof-truss lies at rest under the forces $+x$, $+y$, and -2, with arms $+x$, $-y$, and $+8$. Find the values of x and y.

The equations here needed are $x+y=2$, and $x^2-y^2=16$. Such equations will not be denied an algebraic value, even by the ultra-formalist.

From now on formal problems of any of the customary algebraic types of linear, or linear and quadratic, simultaneous equations may be taken up with understanding on the pupil's part. The work will be backed up with a feeling on the part of the learner that such problems are called for by matters that have a modern meaning and a real use, at least; and this much cannot be said to the credit of the work which the high-school pupil of algebra is commonly called upon to do. High-school pupils work much better under a *mixed* than under a *pure* faith in the ultimate usefulness of what they are required to do.

Almost everyone of the foregoing equations might profitably be given a graphical treatment. It was the purpose of this paper to show at least one rich and easily accessible field to draw upon for material to vitalize and to conceptualize the equations and functions that Mr. Moore would treat very extensively by the aid of cross-ruled paper. It is believed that enough has already been given to accomplish this purpose. If there are those who still doubt the value of such work as is here advocated, let them allow their doubts and skepticism to be dissipated by the test of practical trial!